A compromise between Majority Judgement and Range Voting

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Majority Judgement

Michel BALINSKI, Rida LARAKI: MAJORITY JUDGEMENT

- The Majority Judgement (2007)
  http://ceco.polytechnique.fr/jugement-majoritaire.html
- A theory of measuring, electing and ranking
- Election by Majority Judgement: Experimental evidence

A proposal for voting in political elections
by means of linguistic assessments

\[ \text{median} + \text{breaking ties} \]

Range/Utilitarian Voting

Warren D. Smith: RANGE VOTING

- Range voting (2000)

Claude Hillinger: UTILITARIAN VOTING

- Voting and the cardinal aggregation of judgments
  SEMECON, University of Munich (2004)
- The case for utilitarian voting
  Department of Economics, University of Munich, Discussion paper 2005-11 (2005)

Two proposals for voting in political elections
by means of numerical scales

\[ \text{arithmetic mean} \]

Criticisms on Majority Judgement

  http://rangevoting.org/MedianVrange.html
- D. S. Felsenthal, M. Machover: The Majority Judgment voting procedure: A critical evaluation
  Forthcoming in Homo Oeconomicus
  Computational Intelligence in Decision and Control, World Scientific, Singapore, pp. 531-536
- J. L. García-Lapresta, M. Martínez-Panero (2009): Linguistic-based voting through centered OWA operators
  Forthcoming in Fuzzy Optimization and Decision Making
  Forthcoming
Majority Judgement versus Range Voting

**Majority Judgement**
- It uses the median as aggregation operator
- Breaking ties $\rightarrow$ a lot of cases

**Range Voting**
- It does not use linguistic information but numerical values
- It uses the arithmetic mean as aggregation operator
- It does not need to break ties

Ballot used in the Orsay experiment

**Bulletin de vote du « jugement majoritaire »**

Pour présider la France, ayant pris tous les éléments en compte, je juge en conscience que ce candidat serait :

- Très Bien
- Bien
- Assez Bien
- Passable
- Insuffisant
- À Rejeter

Cochez une seule mention dans la ligne de chaque candidat. Ne pas cocher une mention dans la ligne d’un candidat revient à le Rejeter.

### The Orsay experiment: Official, MJ and RV rankings

<table>
<thead>
<tr>
<th>CANDIDATE</th>
<th>OFFICIAL</th>
<th>MJ</th>
<th>RV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Royal</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Sarkozy</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Bayrou</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Le Pen</td>
<td>4</td>
<td>12</td>
<td>11</td>
</tr>
<tr>
<td>Besancenot</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Villiers</td>
<td>6</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>Voynet</td>
<td>7</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Buffet</td>
<td>8</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Bové</td>
<td>9</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Laguiller</td>
<td>10</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Nihous</td>
<td>11</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Schivardi</td>
<td>12</td>
<td>11</td>
<td>12</td>
</tr>
</tbody>
</table>

### Notation

- $V = \{1, \ldots, m\}$ set of voters ($m \geq 2$)
- $X = \{x_1, \ldots, x_n\}$ set of alternatives ($n \geq 2$)
- $L = \{l_1, \ldots, l_g\}$ ordered set of linguistic terms ($g \geq 2$)
  - $l_1 < \cdots < l_g$

### Example

- $l_1$ to reject
- $l_2$ poor
- $l_3$ acceptable
- $l_4$ good
- $l_5$ very good
- $l_6$ excellent
A profile is a matrix $m \times n$ with coefficients in $L$

$$
\begin{pmatrix}
a_1^1 & \cdots & a_1^j & \cdots & a_1^n \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
a_i^1 & \cdots & a_i^j & \cdots & a_i^n \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
a_m^1 & \cdots & a_m^j & \cdots & a_m^n
\end{pmatrix}
$$

where $a_{ij} \in L$ is the assessment that voter $i$ assigns to $x_j$

- $P$ set of profiles

The middlemost condition in small electorales

Remark (after Galton, 1907)

If $l(x_j) \in L$ fulfills the middlemost condition, this social grade cannot be objectionable by an absolute majority of voters for being either too high or too low

Remark

There always exists $l(x_j) \in L$ for each $x_j \in X$ verifying the middlemost condition, but such a grade might not be necessarily unique

Notation

$L(x_j)$ set of terms satisfying the middlemost condition

- Balinski – Laraki proposal: $l(x_j) = \min L(x_j)$
  In large electorates usually $|L(x_j)| = 1$

The middlemost condition in small electorales

Majority Judgement

$$
\begin{pmatrix}
a_1^1 & \cdots & a_1^j & \cdots & a_1^n \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
a_i^1 & \cdots & a_i^j & \cdots & a_i^n \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
a_m^1 & \cdots & a_m^j & \cdots & a_m^n
\end{pmatrix}
\rightarrow (l(x_1), \ldots, l(x_j), \ldots, l(x_n))
$$

$l(x_j) = f(a_1^j, \ldots, a_m^j)$ $j = 1, \ldots, n$

Middlemost condition (Galton, 1907)

$l(x_j) \in L$ must satisfy

$\#\{i \in V \mid a_i^j \geq l(x_j)\} \geq \frac{m}{2}$ and $\#\{i \in V \mid a_i^j \leq l(x_j)\} \geq \frac{m}{2}$

Majority Judgement $\rightarrow f$ median

Our adjustment

$$
l(x_j) = \begin{cases} 
\text{median } L(x_j) & \text{if } |L(x_j)| \text{ is odd} \\
\text{median } L(x_j) \setminus \{\max L(x_j)\} & \text{if } |L(x_j)| \text{ is even}
\end{cases}
$$

Example

<table>
<thead>
<tr>
<th></th>
<th>TR</th>
<th>P</th>
<th>A</th>
<th>G</th>
<th>VG</th>
<th>E</th>
<th>MJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>4+1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td></td>
<td>TR</td>
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<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
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<td>0</td>
<td>0</td>
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<td>A</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>5</td>
<td></td>
<td>G</td>
</tr>
</tbody>
</table>

A median voter could become a kind of dictator
The *middlemost* condition in small electorales

<table>
<thead>
<tr>
<th>Example</th>
<th>TR</th>
<th>P</th>
<th>A</th>
<th>G</th>
<th>VG</th>
<th>E</th>
<th>MJ</th>
<th>Adjustment</th>
</tr>
</thead>
<tbody>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>TR</td>
<td>A</td>
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<tr>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>P</td>
<td>G</td>
</tr>
<tr>
<td>4</td>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>A</td>
<td>G</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>G</td>
<td>VG</td>
</tr>
</tbody>
</table>

The collective grade under our adjustment might not have been assessed by any voter.

Our proposal

- Majority Judgement is very sensitive towards the median voter
- Range Voting is very sensitive towards extreme assessments

J.L. García-Lapresta, J.L., Martínez-Panero, M.
Linguistic-based voting through centered OWA operators
Forthcoming in *Fuzzy Optimization and Decision Making*

- Voters assign a linguistic term to each candidate
- Individual assessments are aggregated by means of centered OWA operators and the 2-tuple approach
- The outcome is a 2-tuple for each candidate: a linguistic term plus a number (for breaking ties)
- Candidates are sorted and ranked

The 2-tuple approach (Herrera – Martínez, 2000)

\[
\langle L \rangle = L \times [-0.5, 0.5]
\]

is the 2-tuple set associated with \(L\).

The function \(\Delta : [1, g] \rightarrow \langle L \rangle\) is given by

\[
\Delta(\beta) = (l_h, \alpha) \text{ with } \begin{cases} 
  h = \text{round}(\beta) \\
  \alpha = \beta - h
\end{cases}
\]

where \text{round} assigns to \(\beta\) the integer \(h \in \{1, \ldots, g\}\) closest to \(\beta\).

Example

\[
l_1, l_2, l_3, l_4, l_5, l_6
\]

to reject poor acceptable good very good excellent

\[
\langle L \rangle \equiv [1, 6]
\]

\[
\Delta(3.8) = (\text{good}, -0.2) \quad \Delta(4.3) = (\text{good}, 0.3)
\]

OWA operators (Yager, 1988)

Let \(F_w : \mathbb{R}^m \rightarrow \mathbb{R}\) be the OWA operator associated with the weighting vector \(w = (w_1, \ldots, w_m) \in [0, 1]^m\), such that

\[
\sum_{i=1}^{m} w_i = 1
\]

\[
F_w(\beta_1, \ldots, \beta_m) = w_1 \cdot \beta_1 + \cdots + w_m \cdot \beta_m
\]

where \(\beta_{(i)}\) is the \(i\)-th greatest number of \(\beta_1, \ldots, \beta_m\).
Let \( F_w \) be the OWA operator associated with the weighting vector \( w = (w_1, \ldots, w_m) \in [0, 1]^m \), such that \( \sum_{i=1}^m w_i = 1 \).

We say that \( F_w \) is **centered** if the following two conditions are satisfied:

1. **Symmetry**
   
   \[ w_i = w_{m+1-i} \quad \text{for every} \quad i \in \{1, \ldots, \frac{m+1}{2}\} \]

2. **Decaying**

   \[ w_i \leq w_j \quad \text{whenever} \quad i < j \leq \left \lfloor \frac{m+1}{2} \right \rfloor \quad \text{or} \quad i > j \geq \left \lfloor \frac{m+1}{2} \right \rfloor \]

Our proposal

\[ \pi : L \longrightarrow \{1, \ldots, g\} \] is defined by \( \pi(h) = h \) for \( h = 1, \ldots, g \)

**Definition**

Let \( F_w \) the centered OWA operator associated with the weighting vector \( w = (w_1, \ldots, w_m) \)

The mapping \( G_w : P \longrightarrow \langle L \rangle \) is defined by

\[ G_w(P) = (v(x_1), \ldots, v(x_n)) \]

where \( v(x_j) = \Delta(F_w(\pi(a_j^1), \ldots, \pi(a_j^m))) \)

is the collective assessment on \( x_j \)

Majority Judgement and Range Voting as \( F_w \)-procedures

If \( m \) is odd, then Majority Judgement is the \( F_w \)-procedure corresponding to

\[ w_i = \begin{cases} 1 & \text{if} \quad i = \frac{m+1}{2} \\ 0 & \text{otherwise} \end{cases} \]

Range Voting is the \( F_w \)-procedure corresponding to

\[ w_1 = \cdots = w_m = \frac{1}{m} \]
Example 1

\[ V = \{1, \ldots, 9\}, \ X = \{x_1, x_2\}, \ L = \{l_1, \ldots, l_7\} \]

\[
\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\hline
x_1 & l_4 & l_7 & l_7 & l_3 & l_3 & l_3 & l_3 & l_3 \\
x_2 & l_5 & l_6 & l_6 & l_6 & l_1 & l_1 & l_1 & l_1 \\
\end{array}
\]

- Eight out of nine agents prefer \( x_1 \) to \( x_2 \) and only one agent prefers \( x_2 \) to \( x_1 \)
- Under Majority Judgement \( x_2 \) defeats \( x_1 \) because the median of the assessments are \( l_5 \) and \( l_4 \), respectively
- Under a \( F_w \)-procedure
  \[ x_1 \succ x_2 \iff w_5 < 0.6 \]
- Under Range Voting \( x_1 \) defeats \( x_2 \) \((w_5 = 0.11 < 0.6)\)

Example 2

\[ V = \{1, \ldots, 5\}, \ X = \{x_1, x_2\}, \ L = \{l_1, \ldots, l_7\} \]

\[
\begin{array}{ccccccc}
1 & 2 & 3 & 4 & 5 \\
\hline
x_1 & l_7 & l_7 & l_7 & l_7 & l_7 & l_1 \\
x_2 & l_6 & l_6 & l_6 & l_6 & l_6 & l_6 \\
\end{array}
\]

- Four out of five agents prefer \( x_1 \) to \( x_2 \) and only one agent prefers \( x_2 \) to \( x_1 \)
- Under Majority Judgement \( x_1 \) defeats \( x_2 \) because the median of the assessments are \( l_7 \) and \( l_6 \), respectively
- Under a \( F_w \)-procedure
  \[ x_1 \succ x_2 \iff w_1 < 0.14 \]
- Under Range Voting \( x_2 \) defeats \( x_1 \) \((w_1 = 0.2 > 0.14)\)

Concluding remarks

- Majority Judgement is not suitable for small electorates (committees)
- Majority Judgement needs a breaking ties process that uses more information than just the median
- Range Voting is very sensitive towards extreme opinions (outliers)
- The proposed voting system is very flexible and allows us to adapt it to each specific scenario