Weighting individual opinions in group decision making

José Luis García-Lapresta

Dep. de Economía Aplicada, Universidad de Valladolid, PRESAD Research Group
Avda. Valle de Esgueva 6, 47011 Valladolid, Spain
lapresta@eco.uva.es
http://www2.eco.uva.es/lapresta

Abstract. In this paper we introduce a multi-stage decision making procedure where decision makers sort the alternatives by means of a fixed set of linguistic categories, each one has associated a numerical score. First we average the scores obtained by each alternative and we consider the associated collective preference. Then, we obtain a distance between each individual preference and the collective one through the Euclidean distance among the individual and collective scoring vectors. Taking into account these distances, we measure the agreement in each subset of decision makers, and a weight is assigned to each decision maker: his/her overall contribution to the agreement. Those decision makers whose overall contribution to the agreement are not positive are expelled and we re-initiate the decision procedure with only the opinions of the decision makers which positively contribute to the agreement. The sequential process is repeated until it determines a final subset of decision makers where all of them positively contribute to the agreement. Then, we apply a weighted procedure where the scores each decision maker indirectly assigns to the alternatives are multiplied by the weight of the corresponding decision maker, and we obtain the final ranking of the alternatives.

1 Introduction

When a group of decision makers have to decide a collective ranking of a set of alternatives, usually they rank the alternatives and then an aggregation procedure is applied for generating the collective order. If the number of alternatives is high, then decision makers can have difficulties in the task of ranking feasible alternatives. According to Dummett [7]: "If there are, say, twenty possible outcomes, the task of deciding the precise order of preference in which he ranks them may induce a kind of psychological paralysis in the voter".

In order to facilitate decision makers to arrange the alternatives, we propose that decision makers sort the alternatives within a small set of linguistic categories (for instance, *excellent*, *very good*, *good*, *regular*, *bad* and *very bad*). [1]

[1] The use of linguistic information within the decision making framework has been widely used in the literature. See, for instance, Yager [12] and Herrera and Herrera-Viedma [10].

We assign a score to each linguistic category and then a collective score is associated with each alternative by means of the average of the individual scores. Consequently, the alternatives are ordered by the obtained collective scores.

After this first stage, we introduce a distance among individual opinions and the aggregated weak order. Through these distances, we propose an index for measuring the overall contribution to the agreement of each decision maker[2]. Those decision makers whose indices are not positive will be excluded, and we re-initiate the process with only the opinions of the individuals which positively contribute to the agreement. We repeat this procedure, by recalculating the overall indices, until obtaining a final subset of decision makers where all of them positively contribute to the agreement. Then, we weight the scores that decision makers (indirectly) assign to the alternatives by their overall contribution to the agreement indices, and we obtain the final collective ranking of the alternatives.

Notice that weighting individual opinions with the mentioned indices, decision makers are incentivated to not declare very divergent opinions with respect to the mean opinion. Otherwise, they can be penalized by reducing their influence over the collective ranking or being eliminated of the group.

The paper is organized as follows. Section 2 is devoted to introduce the notation and the main notions needed in the multi-stage decision procedure, which we present in Section 3. Finally, Section 4 includes some concluding remarks.

2 Preliminaries

Let $V = \{v_1, \ldots, v_m\}$ a set of decision makers (or voters) who show their preferences on the pairs of a set of alternatives $X = \{x_1, \ldots, x_n\}$, with $m, n \geq 3$. $\mathcal{P}(V)$ denotes the power set of V $(I \in \mathcal{P}(V) \Leftrightarrow I \subseteq V)$. Linear orders are binary relations satisfying reflexivity, antisymmetry and transitivity, and weak orders (or complete preorders) are complete and transitive binary relations. With $|I|$ we denote the cardinal of I.

We consider that each decision maker classifies the alternatives within a set of *linguistic categories* $\mathcal{L} = \{l_1, \ldots, l_p\}$, with $p \geq 2$, linearly ordered $l_1 > \cdots > l_p$. The *individual assignment* of the decision maker v_i is a mapping $C_i : X \longrightarrow \mathcal{L}$ which assigns a linguistic category $C_i(x_u) \in \mathcal{L}$ to each alternative $x_u \in X$.

Associated with C_i, we consider the weak order R_i defined by $x_u \, R_i \, x_v$ if $C_i(x_u) \geq C_i(x_v)$. With P_i and I_i we denote, respectively, the asymmetric (strict preference) and symmetric (indifference) relations associated with R_i, i.e., $x_u \, P_i \, x_v$ whenever not $x_v \, R_i \, x_u$, and $x_u \, I_i \, x_v$ whenever $x_u \, R_i \, x_v$ and $x_v \, R_i \, x_u$.

It is important to note that decision makers are not totally free in declaring their preferences. They have to adjust their opinions to the set of linguistic categories, so the associated weak orders depend on the way they sort the alternatives within the fixed scheme provided by $\mathcal{L}$. Even more, given a weak order

[2] The use of metrics for aggregating individual preferences has been analyzed in the literature by many authors (see, for instance, Kemeny [11], Cook and Seiford [5, 6], Armstrong, Cook and Seiford [1] and Cook, Kress and Seiford [4]).

R_i with no more than p equivalence classes, it is possible to define different individual assignments. For instance, given the weak order $x_1 \, I_i \, x_2 \, P_i \, x_3 \, P_i \, x_4 \, I_i \, x_5$, for $p = 4$ we can associate the assignment: $C_i(x_1) = C_i(x_2) = l_1$, $C_i(x_3) = l_2$ and $C_i(x_4) = C_i(x_5) = l_4$; but also $C_i(x_1) = C_i(x_2) = l_1$, $C_i(x_3) = l_2$ and $C_i(x_4) = C_i(x_5) = l_3$; and so on.

A *profile* is a vector $\mathbf{C} = (C_1, \dots, C_m)$ of individual assignments. We denote by $\mathcal{C}$ the set of profiles.

We assume that every linguistic category $l_k \in \mathcal{L}$ has associated a score $s_k \in \mathbb{R}$ in such a way that $s_1 \geq s_2 \geq \cdots \geq s_p$ and $s_1 > s_p = 0$. For the decision maker v_i, let $S_i : X \longrightarrow \mathbb{R}$ be the mapping which assigns the score to each alternative, $S_i(x_u) = s_k$ whenever $C_i(x_u) = l_k$. The *scoring vector of* v_i is $(S_i(x_1), \dots, S_i(x_n))$.

Naturally, if $s_i > s_j$ for all $i, j \in \{1, \dots, p\}$ such that $i > j$, then each linguistic category is univoquely determined by its associated score. Thus, given the scoring vector of a decision maker we directly know the way this individual sort the alternatives. Although linguistic categories are equivalent to decreasing sequences of scores, there exist clear differences from a behavioral point of view.

Example 1. Consider three decision makers who sort the alternatives of $X = \{x_1, , \dots, x_9\}$ according to the set of linguistic categories $\mathcal{L} = \{l_1, \dots, l_6\}$ and the associated scores given in Table 1.

Table 1. Linguistic categories

$\mathcal{L}$	Meaning	Score
l_1	Excellent	$s_1 = 8$
l_2	Very good	$s_1 = 5$
l_3	Good	$s_1 = 3$
l_4	Regular	$s_1 = 2$
l_5	Bad	$s_1 = 1$
l_6	Very bad	$s_1 = 0$

In Table 2 we include the way of decision makers sort the alternatives within the set of linguistic categories.

It seems reasonable to assign as collective score $S(x_u)$, for each alternative $x_u \in X$, the average of the individual scores:

$$S(x_u) = \frac{1}{m} \sum_{i=1}^{m} S_i(x_u).$$

Taking into account the *average collective scoring vector*, $(S(x_1), \dots, S(x_n))$, we define the *average collective weak order* on X:

$$x_u \, R \, x_v \;\Leftrightarrow\; S(x_u) \geq S(x_v).$$

Table 2. Sorting alternatives

	R_1	R_2	R_3
l_1	x_1	x_3	
l_2	$x_3\ x_6$	x_8	$x_1\ x_2\ x_5$
l_3	$x_2\ x_4\ x_5\ x_8$	$x_4\ x_6$	$x_4\ x_6$
l_4	x_7	$x_1\ x_9$	x_3
l_5	x_9	x_5	$x_7\ x_8$
l_6		$x_2\ x_7$	x_9

Following Example 1, in Table 3 we show the individual and collective scores obtained by each alternative.

Table 3. Scores

	S_1	S_2	S_3	S
x_1	8	2	5	5
x_2	3	0	5	2.666
x_3	5	8	2	5
x_4	3	3	3	3
x_5	3	1	5	3
x_6	5	3	3	3.666
x_7	2	0	1	1
x_8	3	5	1	3
x_9	1	2	0	1

In Table 4 we show the collective preference provided by the weak order R.

If we compare the collective preference with the individual ones in Example 1, it is clear that there exist some differences. In order to have some information about the agreement in each subset of decision makers, we firstly introduce a distance between pairs of preferences (scoring vectors). Since the arithmetic mean minimizes the sum of distances to individual values with respect to the Euclidean metric, it seems reasonable to use this metric for measuring the distance among scoring vectors.

Definition 1. *Let* $(S(x_1), \ldots, S(x_n)), (S'(x_1), \ldots, S'(x_n))$ *be two individual or collective scoring vectors. We define the distance between these vectors by means of the Euclidean metric:*

$$d(S, S') = \sqrt{\sum_{u=1}^{n} (S(x_u) - S'(x_u))^2}.$$

Table 4. Collective order

	x_1		x_3	
		x_6		
x_4		x_5		x_8
		x_2		
	x_7		x_9	

Taking into account Example 1, the distances among the individual opinions and the collective preference are given by:

$$d(S_1, S) = 3.448 < d(S_3, S) = 4.887 < d(S_2, S) = 5.962\,. \qquad (1)$$

In next section we introduce an index which measures the overall contribution to the agreement for each decision maker. By means of these measures, we modify the initial group decision procedure for priorizating consensus[3].

3 The multi-stage decision making procedure

In order to introduce our multi-stage group decision making procedure, we first consider a specific agreement measure which is based on the distances among individual and collective scoring vectors in each subset of decision makers.

We note that Bosch [2] introduced a general concept of *consensus measure* within the class of linear orders by assuming three axioms: Unanimity, Anonymity (symmetry with respect to decision makers) and Neutrality (symmetry with respect to alternatives).

Definition 2. *The mapping* $\mathcal{M} : \mathcal{C} \times \mathcal{P}(V) \longrightarrow [0,1]$ *defined by*

$$\mathcal{M}(\mathbf{C}, I) = \begin{cases} 1 - \dfrac{\sum\limits_{v_i \in I} d(S_i, S)}{|I|\, s_1 \sqrt{n}}, & \textit{if } I \neq \emptyset, \\ 0, & \textit{if } I = \emptyset \end{cases}$$

is called overall agreement measure.

We note that $s_1 \sqrt{n}$ is the maximum distance among scoring vectors, clearly between $(S(x_1), \ldots, S(x_n)) = (s_1, \ldots, s_1)$ and $(S'(x_1), \ldots, S'(x_n)) = (0, \ldots, 0)$:

$$d(S, S') = \sqrt{n\, s_1^2} = s_1 \sqrt{n}.$$

[3] Along the paper we do not talk about consensus, but about agreement. The reason is that consensus has different meanings. One of them is related to an interactive and sequential procedure where decision makers have to change their preferences in order to improve the agreement. Usually, a moderator advise decision makers to modify some opinions (see, for instance, Eklund, Rusinowska and de Swart [8]).

Then, $\mathcal{M}(\boldsymbol{C}, I) \in [0,1]$, for every $(\boldsymbol{C}, I) \in \mathcal{C} \times \mathcal{P}(V)$.

It is important to note that $\mathcal{M}(\boldsymbol{C}, V) = 1$ if and only if $C_1 = \cdots = C_m$; in other words, $\mathcal{M}(\boldsymbol{C}, V) = 1$ if and only if all the decision makers share the same assignment (Unanimity).

The problem of determine the minimum agreement (or total disagreement) presents more difficulties, because in the case of more than 2 decision makers agreement and disagreement are not symmetric notions (see Bosch [2]).

It is easy to see that our overall agreement measure satisfies the other axioms of Bosch [2], Anonymity and Neutrality.

We now introduce an index which measures the overall contribution to the agreement of each voter with respect to a fixed profile, by adding up the marginal contributions to the agreement in all the subsets of decision makers.

Definition 3. *The* overall contribution to the agreement *of decision maker v_i with respect to a profile $\mathbf{C} \in \mathcal{C}$ is defined by:*

$$w_i = \sum_{I \subseteq V} \Big(\mathcal{M}(\mathbf{C}, I) - \mathcal{M}(\mathbf{C}, I \setminus \{v_i\}) \Big).$$

Obviously, if $v_i \notin I$, then $\mathcal{M}(\boldsymbol{C}, I) - \mathcal{M}(\boldsymbol{C}, I \setminus \{v_i\}) = 0$. If $w_i > 0$, we say that decision maker v_i positively contributes to the agreement; and if $w_i < 0$, we say that decision maker v_i negatively contributes to the agreement.

We now introduce a new collective preference by weighting the scores which decision makers (indirectly) assign to alternatives with the corresponding overall contribution to the agreement indices.

Definition 4. *The* collective weak order *associated with the weighting vector $\boldsymbol{w} = (w_1, \ldots, w_m)$, $R^{\boldsymbol{w}}$, is defined by*

$$x_u \, R^{\boldsymbol{w}} \, x_v \;\Leftrightarrow\; S^{\boldsymbol{w}}(x_u) \geq S^{\boldsymbol{w}}(x_v),$$

where

$$S^{\boldsymbol{w}}(x_u) = \frac{1}{m} \sum_{i=1}^{m} w_i \cdot S_i(x_u).$$

Consequently, we prioritize the decision makers in order of their contribution to the agreement (see Cook, Kress and Seiford [4]).

Notice that the average collective weak order is just the collective weak order associated with the weighting vector $\boldsymbol{w} = (1, \ldots, 1)$.

Example 2. Following Example 1 and the overall contributions to the agreement introduced in Definition 3, we obtain $w_1 = 0.670$, $w_2 = 0.557$ and $w_3 = 0.605$. If we apply these weights in the collective decision procedure of Definition 4, then the opinion of the first decision maker counts $w_1/w_2 = 1.203$ times the opinion of the second one; $w_1/w_3 = 1.107$ times the opinion of the second one; and the opinion of the third decision maker counts $w_3/w_2 = 1.087$ times the opinion of the second one.

In Table 5 we show the initial collective scores given in Table 3 and the new collective scores after we weight the opinions of the decision makers with the overall contributions to the agreement. We also include the ratio between the new collective scores, $S^{\boldsymbol{w}}$, and the initial collective scores, S. These differences are due to the individual contributions to the agreement. It is important to note that in the new version of the decision procedure there are not ties.

Table 5. New collective scores

	S	$S^{\boldsymbol{w}}$	$S^{\boldsymbol{w}}/S$
x_1	5	3.168	0.633
x_2	2.666	1.679	0.629
x_3	5	3.006	0.601
x_4	3	1.833	0.611
x_5	3	1.865	0.621
x_6	3.666	2.280	0.621
x_7	1	0.648	0.648
x_8	3	1.800	0.600
x_9	1	0.594	0.594

According to the obtained weights, the new version of the decision procedure linearly order the alternatives, by means of $R^{\boldsymbol{w}}$, in the following way:

$$x_1,\ x_3,\ x_6,\ x_5,\ x_4,\ x_8,\ x_2,\ x_7,\ x_9. \tag{2}$$

Since negative values of w_i could artificially alter the outcomes of $R^{\boldsymbol{w}}$, we consider the weighting vector $\boldsymbol{w}' = (w'_1, \ldots, w'_m)$, where $w'_i = \max\{w_i, 0\}$. This problem is now analyzed through an example.

3.1 An illustrative example

Consider four decision makers who sort the alternatives of $X = \{x_1, , \ldots, x_9\}$ according to the set of linguistic categories $\mathcal{L} = \{l_1, \ldots, l_6\}$ and the associated scores given in Table 1. Table 6 contains the way these decision makers rank the alternatives. In Table 7 we show the individual and collective scores obtained by each alternative, and Table 8 includes the collective preference provided by the weak order R.

The overall contributions to the agreement are $w_1 = 0.387$, $w_2 = 0.151$, $w_3 = -0.204$ and $w_4 = 0.197$. According to these indices, the weighted decision procedure (Definition 4) linearly order the alternatives in the following way:

$$x_4,\ x_5,\ x_3,\ x_1,\ x_2,\ x_7,\ x_8,\ x_6,\ x_9. \tag{3}$$

Table 6. Sorting alternatives

	R_1	R_2	R_3	R_4
l_1	x_3	x_1	$x_2\ x_6\ x_9$	x_5
l_2	$x_1\ x_2\ x_4$	x_4	$x_3\ x_7\ x_8$	x_4
l_3	x_5	x_5	x_1	$x_2\ x_3\ x_7\ x_8$
l_4	$x_6\ x_7$	$x_7\ x_8$		x_9
l_5	$x_8\ x_9$	$x_2\ x_3\ x_6$	x_4	x_1
l_6		x_9	x_5	x_6

Table 7. Scores

	S_1	S_2	S_3	S_4	S
x_1	5	8	3	1	4.25
x_2	5	1	8	3	4.25
x_3	8	1	5	3	4.25
x_4	5	5	1	5	4
x_5	3	3	0	8	3.5
x_6	2	1	8	0	2.75
x_7	2	2	5	3	3
x_8	1	2	5	3	2.75
x_9	1	0	8	2	2.75

Since the third decision maker negatively contributes to the agreement, then his/her associated scores are multiplied by a negative weight. In order to avoid this undesirable effect, we will consider non negative weights $w'_i = \max\{w_i, 0\}$: $w'_1 = 0.387$, $w'_2 = 0.151$, $w'_3 = 0$ and $w'_4 = 0.197$. Applying again the decision procedure, we obtain a new linear order on the set of alternatives:

$$x_3,\ x_4,\ x_1,\ x_5,\ x_2,\ x_7,\ x_8,\ x_6,\ x_9. \tag{4}$$

Note that x_3 is ranked in the third position in (3) and it is the first alternative in (4). Since in (3), $S_3(x_3) = 5$ has been multiplied by the negative weight $w_3 = -0.204$, this alternative has been penalized. However, in (4) the opinion of the third decision maker has not been considered. This fact joint with the first decision maker, with the highest weight $w_1 = 0.387$, ranks x_3 at the first alternative, induce that this alternative reaches the top position.

Although the new ranking (4) is more appropriate than (3) for reflecting the individual opinions, it is important to note that all the calculations have been made taking into account the opinions of the third decision maker. If we think that the third decision maker judgments should not be considered (because his/her divergent opinions with respect to the global opinion), we can start a

Table 8. Collective order

x_1 x_2 x_3
x_4
x_5
x_7
x_6
x_8 x_9

new step of the decision procedure where only the opinions of the rest of the decision makers are taken into account.

Table 9. Sorting alternatives

	R_1	R_2	R_3	R_4
l_1	x_3	x_1	x_2 x_6 x_9	x_5
l_2	x_1 x_2 x_4	x_4	x_3 x_7 x_8	x_4
l_3	x_5	x_5	x_1	x_2 x_3 x_7 x_8
l_4	x_6 x_7	x_7 x_8		x_9
l_5	x_8 x_9	x_2 x_3 x_6	x_4	x_1
l_6		x_9	x_5	x_6

In Table 10 we show the individual and collective scores obtained by each alternative, and Table 11 contains the collective preference provided by the weak order R.

The new overall contributions to the agreement are

$$w_1^{(2)} = 0.583 > w_2^{(2)} = 0.570 > w_4^{(2)} = 0.566,$$

while

$$w_1^{(1)} = w_1 = 0.387 > w_4^{(1)} = w_4 = 0.197 > w_2^{(1)} = w_2 = 0.151.$$

These differences are due to the fact that in the second iteration of the decision procedure the divergent opinions of the third decision maker have not been considered.

According to the weights $w_1^{(2)}$, $w_2^{(2)}$, $w_4^{(2)}$, the new stage of the decision procedure linearly order the alternatives in the following way:

$$x_4,\ x_1,\ x_5,\ x_3,\ x_2,\ x_7,\ x_8,\ x_6,\ x_9. \tag{5}$$

Clearly, there exist important differences among the linear orders provided by (3), (4) and (5). In fact, (3) takes into account the divergent opinions of the

Table 10. Scores

	S_1	S_2	S_4	S
x_1	5	8	1	4.666
x_2	5	1	3	3
x_3	8	1	3	4
x_4	5	5	5	5
x_5	3	3	8	4.666
x_6	2	1	0	1
x_7	2	2	3	2.333
x_8	1	2	3	2
x_9	1	0	2	1

Table 11. Collective order

x_4
x_1 x_5
x_3
x_2
x_7
x_8
x_6 x_9

third decision maker; (4) does not consider the opinions of the third decision maker, but the collective ranking and, consequently, all the weights are based on the opinions of all the decision makers, including that of the divergent third decision maker; finally, (5) totally excludes the opinions of the third decision maker.

3.2 Scheme of the multi-stage group decision procedure

In 3.1 we have analyzed through examples how aggregate individual opinions by considering the overall contributions to the agreement. We now present the considered multi-stage decision procedure in a general and precise way.

1. Decision makers $V = \{v_1, \ldots, v_m\}$ sort the alternatives of $X = \{x_1, \ldots, x_n\}$ according to the linguistic categories of $\mathcal{L} = \{l_1, \ldots, l_p\}$. Then, we obtain individual weak orders $R_1, \ldots, R_m$ which rank the alternatives within the fixed set of linguistic categories.
2. Taking into account the scores $s_1, \ldots, s_p$ associated with $l_1, \ldots, l_p$, a score is assigned to each alternative for every decision maker: $S_i(x_u)$, $i = 1, \ldots, m$, $u = 1, \ldots, n$.

3. We aggregate the individual opinions by means of collective scores which are defined as the average of the individual scores:

$$S(x_u) = \frac{1}{m} \sum_{i=1}^{m} S_i(x_u)$$

and we rank the alternatives through the collective weak order R:

$$x_u \, R \, x_v \;\Leftrightarrow\; S(x_u) \geq S(x_v).$$

4. We calculate the overall contributions to the agreement (Definition 3) for all the decision makers: $w_1, \ldots, w_m$.
   (a) If $w_i \geq 0$ for every $i \in \{1, \ldots, m\}$, then we obtain the new collective scores by:

   $$S^{\boldsymbol{w}}(x_u) = \frac{1}{m} \sum_{i=1}^{m} w_i \cdot S_i(x_u)$$

   and we rank the alternatives by means of the collective weak order $R^{\boldsymbol{w}}$:

   $$x_u \, R^{\boldsymbol{w}} \, x_v \;\Leftrightarrow\; S^{\boldsymbol{w}}(x_u) \geq S^{\boldsymbol{w}}(x_v).$$

   (b) Otherwise, we eliminate those decision makers whose overall contributions to the agreement are negative. We now initiate the decision procedure for the remaining decision makers $V^+ = \{v_i \in V \mid w_i \geq 0\}$.

4 Concluding remarks

Usually decision makers have difficulties to rank order a high number of alternatives. In order to facilitate this task, we have considered a mechanism where decision makers sort alternatives through a small set of linguistic categories. We associate a score to each linguistic category and then we aggregate the individual opinions by means of the average of the individual scores, providing a collective weak order on the set of alternatives. Then we assign an index to each decision maker which measures his/her overall contribution to the agreement. Taking into account these indices, we weight individual scores and we obtain a new collective ranking of alternatives after excluding the opinions of those decision makers whose overall contributions to the agreement are not positive. The new collective ranking of alternatives provides the final decision.

Since overall contribution to the agreement indices (which multiply individual scores) usually are irrational numbers, it is unlikely that the weighted procedure provides ties among alternatives.

Since the proposed decision procedure penalizes those individuals that are far from consensus positions, this fact incentives decision makers to moderate their opinions. Otherwise, they can be excluded or their opinions can be underestimated. However, it is worth emphasizing that our proposal only requires a single judgement to each individual about the alternatives.

We can generalize our group decision procedure by considering different aggregation operators (see Fodor and Roubens [9] and Calvo, Kolesárova, Komorníková and Mesiar [3]) for obtaining the collective scores. Another way of generalization consists in measuring distances among individual and collective scoring vectors by means of different metrics.

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